Reflection and absorption of QWs irradiated by light pulses in a strong magnetic field

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It has been shown that the non-sinusoidal character oscillations appear in the transmitted, reflected and absorbed light fluxes when light pulses irradiate a semiconductor quantum well (QW), containing a set of a large number of the equidistant energy levels of electronic excitations. The oscillation amplitude is comparable to the flux values for the short pulses, duration of which $\gamma_l^{-1} \leq \hbar/\Delta E$. A damping echo of the exciting pulse appears through the time intervals $2\pi\hbar/\Delta E$ in the case of the very short light pulses $\gamma_l^{-1} \ll \hbar/\Delta E$. Symmetrical and asymmetrical pulses with a sharp front have been considered. Our theory is applicable for the narrow QWs in a strong magnetic field, when the equidistant energy levels correspond to the electron-hole pairs (EHP) with different Landau quantum numbers.

I. INTRODUCTION

A response of different physical systems on a pulse light irradiation is a subject of strong interest in the past decade [1,2,3,4]. The availability of short-pulse techniques and commercial devices made it possible to investigate coherent phenomena in the processes of interaction between the light and elementary excitations in various systems, thus providing valuable information on their excitation spectra and the mechanisms of relaxation.

A row of theoretical and experimental investigations is devoted to the elaborate study of the Wannier-Mott excitons in the bulk crystals and semiconductor QWs with the help of the time resolved scattering (TRS), because just the existence of the discrete energy levels determines the most interesting results obtainable by the TRS. It is well known that a couple of the closely disposed energy levels demonstrate a new effect: The sinusoidal beatings appear in reflected and transited pulses on a frequency corresponding to the energy distance between the energy levels (see, for instance, Ref. [1]).

In this paper we examine theoretically light pulse reflection and absorption by the semiconductor QWs in a strong magnetic field (SMF) directed perpendicularly to the QW plane xy. In such a case, excitations in the QW are characterized by the quasi-momentum $\mathbf{K}_\perp$ in the xy plane, because the system is homogeneous in this plane. If a movement along the z axis is a finite one, the rest of the indexes of the excitations are discrete ones. When the excitations are created by light, the condition $\mathbf{K}_\perp = \kappa_\perp$, where $\kappa_\perp$ is the light wave vector projection on the xy plane, is satisfied. We examine the case of the normal irradiation when $\mathbf{K}_\perp = \kappa_\perp = 0$. Under this condition the spectrum is always discrete for the finite movement along the z axis. [5]

If the light pulse carrier frequency ω_l exceeds slightly the QW semiconductor energy gap E_g, then the created by light excitations are the electron-hole pairs (EHP), which are characterized by the indexes $n_e = n_h = n,\ l_e,\ l_h$, where $n_e(n_h)$ is the Landau quantum number of an electron (hole), $l_e(l_h)$ is the electron (hole) size-quantization (along the z axis) quantum number. In the case of the infinitely deep QW $l_e = l_h$, but we will not restrain our consideration by this restriction. The excitation energy, measured from the ground state energy, is equal

$$E_{\xi_0} = E_g + \varepsilon^e_{l_e} + \varepsilon^h_{l_h} + (n + 1/2)\hbar\Omega_\mu, \qquad (1)$$

where ξ_0 is the set of the indexes $\mathbf{K}_\perp = 0,\ n,\ l_e,\ l_h;\ \ \varepsilon^e_{l_e}\ (\varepsilon^h_{l_h})$ is the electron (hole) energy on the size-quantized energy level l (see, for instance, Ref. [6]), $\Omega_\mu = |e|H/\mu c,\ \mu = m_e m_h/(m_e + m_h),\ m_e\ (m_h)$ is the electron (hole) effective mass. It follows from Eq. (1) that the excitation energies are equidistant for the fixed quantum numbers l_e and l_h. Generally speaking, the level equidistancy is broken if one takes into account the Coulomb electron-hole interaction, i. e. the excitonic effect. But the Coulomb interaction is a weak perturbation, if the conditions

$$d << a, \quad a_H << a,$$

(d is the QW width, a is the Wannier-Mott exciton radius in the magnetic field absence, $a_H = (c\hbar/|e|H)^{1/2}$ is the magnetic length) are satisfied, and leads only to the small shifts of the energy levels Eq. (1). [7] We suppose that the QWs are narrow and that magnetic fields are strong enough to ignore the excitonic effects. An equidistancy violation may be due to the semiconductor band non-parabolicity , but we suppose that the non-parabolicity is weak in the vicinity of the bottoms of the conduction and valence bands.

Let us suppose that a QW with a system of the equidistant energy levels is irradiated by the light pulse with a carrier frequency ω_l. Let us admit that the carrier frequency ω_l is alongside the resonance with one of the energy levels. Then two variants are possible: One can ignore the influence of the rest levels or take into account some number of the neighbor energy levels. The choice between the variants depends on the pulse form and duration, i. e. from the pulse frequency spectrum.

Some ladder-type structure has been predicted in Refs. [8,9] for the reflected and transmitted pulses for the QW in the SMF irradiated by the sharply asymmetrical light pulse with a sharp front, which corresponds to the second variant mentioned above. The first variant can be realized, for instance, in the case of a symmetrical pulse under condition $\gamma_l << \Delta\omega$, $\Delta\omega$ is the distance between the neighbor levels, γ_l^{-1} is the pulse duration, , γ_l is the frequency dispersion.

A large number of the theoretical investigations have been devoted to the examination of the QW electronic system response in the cases of one or two excited energy levels. We have to stress that a violation of the translation invariance in the direction perpendicular to the QW plane leads to the radiative broadening γ_r of the energy levels [10,11]. In the case of the high quality QWs the radiative broadening may be comparable to or even exceed the contributions of other relaxation mechanisms. This physical situation demands an adequate theoretical description, where one has to take into account the high orders of the electron-electromagnetic field (EMF) interaction (see Refs. [8,9,10,11,12,13,14,15,16,17,18,19,20,21]. In these investigations a transparent QW is considered, i. e. it is supposed, that light absorption and reflection are due to the existence of one or two excited energy level. In the case when

$$\gamma_r << \gamma, \tag{2}$$

where γ is the non-radiative inverse lifetime of the electronic excitations, the perturbation theory is applicable and the lowest order on the electron-EMF interaction is enough when the response on the monochromatic or pulse irradiation is investigated. [††]

Under condition Eq. (2) the induced EMFs on the left- and on the right hand side of the QW are small in comparison with the exciting EMFs; in the case of monochromatic irradiation the reflection ($\mathcal{R}$) and absorption ($\mathcal{A}$) coefficients are small in comparison with unity, and the transmitted pulse differs slightly from the exciting pulse form. However, even in this situation the very interesting experimental results are obtained: Delaying of a short pulse on the times of order γ^{-1} had been seen in the transmitted light, and the sinusoidal beatings on the frequencies $\Delta E/\hbar$ (where ΔE is the distance between the energy levels) had been seen (see, for instance, Ref. [1]).

In the opposite case

$$\gamma_r \geq \gamma, \tag{3}$$

the induced EMFs are comparable to the exciting EMFs, the coefficient $\mathcal{R}$ can be close to unity, the coefficient $\mathcal{A}$ is close to 1/2. The results of the QW irradiation by the monochromatic light under condition Eq. (3) in the case of the only excited energy level had been obtained in Refs. [10,11,12,13,18] and in the case of the two excited energy levels in Ref. [20], respectively. The form of the reflected and transmitted pulses closely to the resonance of the carrier frequency with the only energy level in a QW had been found in Ref. [17]. The analytical solution for a non-symmetrical pulse with a sharp front had been found and the numerical calculation results had been obtained for the symmetrical "one-overhyperbolic-cosine" pulse. The EHP radiative lifetimes in QWs in SMFs had been calculated in Refs. [8,20].

In this paper we study a response of a multilevel excitation system in a QW, subjected to the SMF and irradiated by the symmetric light pulse. The results are compared with those for the symmetrical pulse with a sharp front.

II. ELECTRIC FIELDS ON THE RIGHT- , LEFT HAND SIDE FROM A QW IRRADIATED BY LIGHT PULSES.

Let us assume that a time-limited light pulse drops down from the left on a single QW perpendicular to its surface. The electric field of the pulse is

$$\mathbf{E}_0(z,t) = E_0\mathbf{e}_1 e^{-i\omega_l p}\Big\{\Theta(p)e^{-\gamma_{l1}p/2} + [1-\Theta(p)]e^{\gamma_{l2}p/2}\Big\} + c.c., \tag{4}$$

where E_0 is the real amplitude, $\mathbf{e}_l$ is the polarization vector, ω_l is the pulse carrier frequency,

$$p = t - zn/c, \tag{5}$$

n is the refraction index out of the QW , $\Theta(p)$ is the Haeviside function . The Umov-Pointing vector corresponds to the pulse Eq.(4)

$$\mathbf{S}(z,t) = \mathbf{S}_0 P(p), \tag{6}$$

$$\mathbf{S}_0 = \frac{\mathbf{e}_z}{2\pi}\frac{c}{n}E_0^2,$$
$$P(p) = \Theta(p)e^{-\gamma_{l1}p/2} + [1-\Theta(p)]e^{\gamma_{l2}p/2}\}, \tag{7}$$

$\mathbf{e}_z$ is the unit vector along the z axis. Let us make the Fourier transformation of Eq.(4)

$$\mathbf{E}_0(z,t) = E_0\mathbf{e}_1\int_{-\infty}^{\infty} d\omega e^{-i\omega p}D_0(\omega) + c.c., \tag{8}$$

$$D_0(\omega) = \frac{i}{2\pi}\left[(\omega - \omega_l + i\gamma_{l1})^{-1} - (\omega - \omega_l - i\gamma_{l2})^{-1}\right] \quad (9)$$

The pulse is symmetrical one under condition

$$\gamma_{l1} = \gamma_{l2} = \gamma_l. \quad (10)$$

When $\gamma_l \to 0$ it transfers into a monochromatic light wave with the frequency ω_l, and the function $D_0(\omega)$ transfers into the Dirac $\delta(\omega - \omega_l)$-function.

In Refs. [8,9,17] a strongly asymmetrical pulse with a sharp front had been used, when $\gamma_{l2} \to \infty$ and the second term in the braces in the RHS of Eq. (4) vanishes, as well as the second term in the square brackets in the RHS of Eq. (9).

The pulse Eq. (4) is very useful for calculations. Its imperfection is a sharp form of the peak at $t - zn/c = 0$, i. e. the derivative discontinuity of the function $P(p)$ at $p = 0$. However, all the qualitative conclusions of the theory, obtained below, do not change for the smooth pulses.

Let us suppose that the incident waves have the circular polarization, i. e.

$$\mathbf{e}_l = \frac{1}{\sqrt{2}}(\mathbf{e}_x \pm i\mathbf{e}_y), \quad (11)$$

where $\mathbf{e}_x, \mathbf{e}_y$ are the unit vectors along the axis x, y.

Let us consider the QW , the width d of which is much smaller than the light wave length $c/n\omega_l$. Then the electric field $\mathbf{E}_{left(right)}$ on the left (right) hand side of the QW is determined by the expressions [17]

$$\mathbf{E}_{left(right)}(z,t) = \mathbf{E}_0(z,t) + \Delta\mathbf{E}_{left(right)}(z,t), \quad (12)$$

$$\Delta\mathbf{E}_{left(right)}(z,t) = E_0\mathbf{e}_1 \int_{-\infty}^{\infty} d\omega e^{-i\omega(t \pm zn/c)} D(\omega) + c.c., \quad (13)$$

where the upper (lower)sign corresponds to the index "left" ("right"). The frequency partition function $D(\omega)$ is determined as

$$\mathcal{D}(\omega) = -\frac{4\pi\chi(\omega)\mathcal{D}_0(\omega)}{1 + 4\pi\chi(\omega)}, \quad (14)$$

$$\chi(\omega) = \frac{i}{4\pi}\sum_\rho \frac{\gamma_{r\rho}}{2} \times \left[(\omega - \omega_\rho + i\gamma_\rho/2)^{-1} + (\omega + \omega_\rho + i\gamma_\rho/2)^{-1}\right] + \frac{Q(\omega)}{4\pi} - i\frac{I(\omega)}{4\pi}, \quad (15)$$

where the index ρ is the number of the excited state, $\hbar\omega_\rho$ is the excitation energy measured from the ground state energy, $\gamma_{r\rho}(\gamma_\rho)$ is the radiative (non-radiative) inverse lifetime of the excitation ρ, $Q(\omega)$ and $I(\omega)$ determine the contributions to the real and imaginary parts of the function $\chi(\omega)$ due to the unaccounted electronic excitations (for instance, the excitations from the deeper levels than the valence band) and by the lattice excitations.

We will consider, as in Refs. [8,9,10,11,12,13,14,15,16,17,18,20], a i. e. we presume

$$Q(\omega) \simeq 0, \quad I(\omega) \simeq 0, \quad (16)$$

and ignore the second non-resonant term in the square brackets in the RHS of Eq.(15). [‡] Thus, we suppose

$$\chi(\omega) \simeq \frac{i}{4\pi}\sum_\rho \frac{\gamma_{r\rho}}{2}(\omega - \omega_\rho + i\gamma_\rho/2)^{-1}. \quad (17)$$

Substituting Eq. (17) into Eq. (14), we obtain

$$D(\omega) = -\frac{i\sum_\rho(\gamma_{r\rho}/2)(\omega - \omega_\rho + i\gamma_\rho/2)^{-1}D_0(\omega)}{1 + i\sum_\rho(\gamma_{r\rho}/2)(\omega - \omega_\rho + i\gamma_\rho/2)^{-1}}. \quad (18)$$

Substituting Eq. (18) into Eq. (13), we obtain the induced field as a sum of two terms

$$\Delta\mathbf{E}_{right}(z,t) = \Delta\mathbf{E}_1(p) + \Delta\mathbf{E}_2(p). \quad (19)$$

The first is the contribution of the poles of the function $D_0(\omega)$. Applying Eqs. (9) and (13), it is easily to obtain

$$\begin{aligned}\Delta\mathbf{E}_1(p) = {} & -iE_0\mathbf{e}_l e^{-i\omega_l p} \\ & \times \Big\{\Theta(p)e^{-\gamma_{l1}p/2} \\ & \times \frac{\sum_\rho(\gamma_{r\rho}/2)[\omega_l - \omega_\rho + i(\gamma_\rho - \gamma_{l1})/2]^{-1}}{1 + \sum_\rho(\gamma_{r\rho}/2)[\omega_l - \omega_\rho + i(\gamma_\rho - \gamma_{l1})/2]^{-1}} \\ & + [1 - \Theta(p)]e^{\gamma_{l2}p/2} \\ & \times \frac{\sum_\rho(\gamma_{r\rho}/2)[\omega_l - \omega_\rho + i(\gamma_\rho + \gamma_{l2})/2]^{-1}}{1 + \sum_\rho(\gamma_{r\rho}/2)[\omega_l - \omega_\rho + i(\gamma_\rho + \gamma_{l2})/2]^{-1}}\Big\} \\ & + c.c.. \end{aligned} \quad (20)$$

The second term $\Delta\mathbf{E}_2(p)$ is the contribution of the poles of the function

$$\frac{i\sum_\rho(\gamma_{r\rho}/2)(\omega - \omega_\rho + i\gamma_\rho/2)^{-1}}{1 + i\sum_\rho(\gamma_{r\rho}/2)(\omega - \omega_\rho + i\gamma_\rho/2)^{-1}}.$$

In the case of one excited energy level we have one pole $\omega_0 - i(\gamma + \gamma_r)/2$, in the case of two energy levels we have two poles, the positions of which are easily determined.

However, already in the case of three energy levels it is difficult to determine the poles, because one has to solve the third order equation. Therefore to calculate $\Delta\mathbf{E}_2(p)$ in the case of the large number of the energy levels one needs to use an approximation, which is applicable only for the small values $\gamma_{r\rho}$. Constraining with the lowest order term on the electron-EMF interaction and supposing

$$D(\omega) \simeq -4\pi\chi(\omega)D_0(\omega), \quad (21)$$

we obtain the approximate result for the function $\Delta\mathbf{E}_2(p)$

$$\Delta\mathbf{E}_2(p) \simeq -iE_0\mathbf{e}_l\Theta(p)\sum_\rho(\gamma_{r\rho}/2)e^{-i\omega_\rho p-(\gamma_\rho)/2}$$
$$\times\left\{[\omega_l-\omega_\rho+i(\gamma_\rho-\gamma_{l1})/2]^{-1}\right.$$
$$\left.-\ [\omega_l-\omega_\rho+i(\gamma_\rho+\gamma_{l2})/2]^{-1}\right\}+c.c.. \qquad (22)$$

Eq. (20) contains the sum

$$S=\sum_\rho(\gamma_{r\rho}/2)(\omega_l-\omega_\rho+i\bar{\gamma}_\rho/2)^{-1},$$

where $\bar{\gamma}_\rho=\gamma_\rho-\gamma_{l1}$ or $\bar{\gamma}_\rho=\gamma_\rho+\gamma_{l2}$. The real parts of these sums diverge, if the values $\gamma_{r\rho}$ do not depend on ρ. In the case of the equidistant energy levels this divergence is logarithmic one. This divergence is eliminated indeed, but it is very difficult to determine the eliminating mechanism. Thus we will act as follows: Let us write the sum S as

$$S=\sum_\rho{}'(\gamma_\rho/2)(\omega_l-\omega_\rho+i\bar{\gamma}_\rho/2)^{-1}-J(\omega_l), \qquad (23)$$

where the sign $'$ in the sum on ρ means the summing on the limited number of the energy levels. The non-dimensional value $J(\omega_l)$ depends weakly on ω_l, if the frequency set, determined by Eq. (9), is in the resonance with the level group, containing in the first term in the RHS of Eq. (23). Eq. (23) has to be opposed with Eq. (15). The value $J(\omega_l)\simeq J$ is added to the constant $I(\omega)\simeq I$. Thus, we do not know the value J , and in any case, $J<<1$ or $J\geq 1$, the contribution $\Delta\mathbf{E}_1(p)$ to induced electric field damps on times of order γ_l^{-1}, as the follows from Eq. (20). We postpone the question about the divergencies which can appear in the RHS of Eq.(22) until section Y. We will see that the divergencies do not appear in the case of the symmetrical pulses. The expression for the induced field $\Delta\mathbf{E}_{left}(z,t)$ on the left hand side of the QW differs from Eqs. (19)-(22) only by substitution $s=t+zn/c$ instead of $p=t-zn/c$.

III. TRANSMITTED, REFLECTED AND ABSORBED ENERGY FLUXES.

For the sake of brevity we will call the Umov-Pointing vectors as the energy fluxes. The transmitted energy flux, i.e. the flux on the right hand side of the QW, is equal

$$\mathbf{S}_{right}(z,t)=\frac{\mathbf{e}_z}{4\pi}\frac{c}{n}|\mathbf{E}_{right}(z,t)|^2, \qquad (24)$$

the energy flux on the left hand side of the QW, is equal

$$\mathbf{S}_{left}(z,t)=\mathbf{S}(z,t)+\mathbf{S}_{ref}(z,t), \qquad (25)$$

where $\mathbf{S}(z,t)$ is the energy flux of the exciting pulse determined by Eq. (6), $\mathbf{S}_{ref}(z,t)$ is the reflected energy flux, which is equal

$$\mathbf{S}_{ref}(z,t)=-\frac{\mathbf{e}_z}{4\pi}\frac{c}{n}|\Delta\mathbf{E}_{left}(z,t)|^2. \qquad (26)$$

The absorbed energy flux is defined as

$$\mathbf{S}_{abs}(t)=\mathbf{S}_{left}(z=0,t)-\mathbf{S}_{right}(z=0,t) \qquad (27)$$

and equals

$$\mathbf{S}_{abs}=-\frac{\mathbf{e}_z}{2\pi}\frac{c}{n}\mathbf{E}_{right}(z=0,t)\Delta\mathbf{E}(z=0,t), \qquad (28)$$

where

$$\Delta\mathbf{E}(z=0,t)=\Delta\mathbf{E}_{left}(z=0,t)$$
$$=\Delta\mathbf{E}_{right}(z=0,t). \qquad (29)$$

Let us introduce the non-dimensional functions $\mathcal{R}(t),\mathcal{A}(\mathrm{t})$ and $\mathcal{T}(t)$ with the help of the interrelations

$$\mathbf{S}_{ref}(z,t)=-\mathbf{S}_0\mathcal{R}(s),\quad \mathbf{S}_{abs}(t)=\mathbf{S}_0\mathcal{A}(t),$$
$$\mathbf{S}_{right}(z,t)=\mathbf{S}_0\mathcal{T}(p). \qquad (30)$$

It follows from Eq. (27), that

$$\mathcal{R}(t)+\mathcal{A}(t)+\mathcal{T}(t)=P(t). \qquad (31)$$

The values $\mathcal{R}(t)$ and $\mathcal{T}(t)$ are always positive ones, the absorption $\mathcal{A}(t)$ may be positive as well as negative.

Let us consider a system with the arbitrary number of the energy levels irradiated with the pulse Eq. (4). When calculating Eq. (22) we have already used the condition, according to which the parameters $\gamma_{r\rho}$ are the smallest ones among the parameters of our task, and the times $p<<\gamma_{r,\rho}^{-1},\ s<<\gamma_{r,\rho}^{-1}$ have been considered. Now we use an additional condition of a short pulse

$$\gamma_\rho<<\gamma_l \qquad (32)$$

and consider the times

$$p>>\gamma_l^{-1},\quad s>>\gamma_l^{-1}. \qquad (33)$$

It is obvious, that then only the contribution $\Delta\mathbf{E}_2(p)$, containing,according to Eq. (22), $\exp(-\gamma_\rho p/2)$, is preserved in the RHS of Eq. (19). The contribution $\Delta\mathbf{E}_1(p)$, determined in Eq. (20), and the exciting field $\mathbf{E}_0(z,t)$ are small, because they content the factor $\exp(-\gamma_l p/2)$. Thus, under conditions Eqs. (32) and (33) we obtain

$$\mathbf{E}_{right}(z,t)\simeq\Delta\mathbf{E}_2(p),\quad \mathbf{E}_{left}(z,t)\simeq\Delta\mathbf{E}_2(s), \qquad (34)$$

from which it follows, that the transmitted and reflected fluxes are equal in absolute values, i. e.

$$\mathcal{R}(t)\simeq\mathcal{T}(t). \qquad (35)$$

Because on the times of Eq. (33)

$$P(t)\simeq 0,$$

we obtain from Eq. (31)

$$\mathcal{A}(t) = -2\mathcal{R}(t). \tag{36}$$

The negative absorption, which is equal to the double reflection, means that the QW gives back the accumulated energy, radiating it symmetrically by two fluxes on the left- and on the right hand side of the QW. This energy was accumulated as excitations, created during the pulse transmission.

IV. THE EQUIDISTANT ENERGY LEVEL SYSTEM IN A QW IN A SMF.

Under conditions Eqs. (32) and (33) in the case of the symmetrical pulse we obtain from Eqs. (34) and (22)

$$\begin{aligned}\mathbf{E}^{sim}_{right}(z,t) \simeq iE_0\mathbf{e}_l\Theta(s)\sum_\rho(\gamma_{r\rho}/2)e^{-i\omega_\rho p-\gamma_\rho p/2}\\ \times\left\{[\omega_l-\omega_\rho+i(\gamma_\rho-\gamma_l)/2]^{-1}\right.\\ \left.-\ [\omega_l-\omega_\rho+i(\gamma_\rho+\gamma_l)/2]^{-1}\right\}.\end{aligned} \tag{37}$$

In the case of the asymmetrical pulse with the sharp front (see Refs. [8,9,17] we obtain the result of Eq. (37), in which the second term in the braces is absent. One can see easily, that the sum on ρ in the RHS of Eq. (37) does not diverge at the large values of ρ, no matter what the value of the variable p is. This divergence can appear in the case of an asymmetrical pulse.

Let us simplify Eq. (37), supposing that the inverse lifetimes $\gamma_{r\rho}$ and γ_ρ are equal for all the energy levels, i. e.

$$\gamma_{r\rho}\simeq\gamma_r,\quad \gamma_\rho\simeq\gamma, \tag{38}$$

and ignoring the small values γ comparing with γ_l in the square brackets. Then for the reflected flux and the symmetrical pulse we obtain with the help Eq. (26)

$$\begin{aligned}\mathcal{R}^{sim}(s)\simeq\frac{\gamma_r^2\gamma_l^2}{4}e^{-\gamma s}\sum_{\rho,\rho'}[(\omega_l-\omega_\rho)^2+\gamma_l^2/4]^{-1}\\ \times[(\omega_l-\omega_{\rho'})^2+\gamma_l^2/4]^{-1}\cos[(\omega_\rho-\omega_{\rho'})s].\end{aligned} \tag{39}$$

The analogical result for the asymmetrical pulse is as follows

$$\begin{aligned}\mathcal{R}^{asim}(s)\simeq\frac{\gamma_r^2}{4}e^{-\gamma s}\sum_{\rho,\rho'}[(\omega_l-\omega_\rho)^2+\gamma_l^2/4]^{-1}\\ \times[(\omega_l-\omega_{\rho'})^2+\gamma_l^2/4]^{-1}\\ \times\{(\gamma_l/2)(\omega_\rho-\omega_{\rho'})\sin[(\omega_\rho-\omega_{\rho'})s]\\ +[(\omega_l-\omega_\rho)(\omega_l-\omega_{\rho'})+\gamma_l^2/4]\\ \times\cos[(\omega_\rho-\omega_{\rho'})s]\}.\end{aligned} \tag{40}$$

Let us apply Eqs. (39), (40) in the case of the equidistant Landau levels (LLs), corresponding to Eq. (1) for the EHP energy at the fixed size-quantized numbers l_e and l_h for electrons and holes, respectively. The frequencies ω_ρ and ω_l from the RHS of Eqs. (39),(40) are measured from the value

$$E_g/\hbar+\varepsilon^e_{le}+\varepsilon^h_{lh}+\Omega_\mu/2.$$

Then

$$\omega_\rho = n\Omega_\mu,\quad n=0,1,2........ \tag{41}$$

Let us substantiate the suppose Eq. (38) for the QW in a SMF. The radiative lifetime of the EHPs, which is applicable for the heavy holes in GaAs, has been calculated in Ref. [20]. There are two sorts of the EHPs, designated by the indexes I and II. These sorts are distinguished by the values of the interband momentum matrix elements $\mathbf{p}_{cv}$, which are

$$\mathbf{p}^I_{cv}=\frac{1}{\sqrt{2}}p_{cv}(\mathbf{e}_x-i\mathbf{e}_y),\quad \mathbf{p}^{II}_{cv}=\frac{1}{\sqrt{2}}p_{cv}(\mathbf{e}_x+i\mathbf{e}_y). \tag{42}$$

When we use the circular polarizations Eq. (11), every polarization is linked strongly with the EHP sort I or II, because the EHP-light interaction is proportional to the scalar production $\mathbf{e}_l\mathbf{p}_{cv}$ (compare with [‡]). It has been shown [20]that the inverse radiative lifetime of any sort is equal at $\mathbf{K}_\perp=0$

$$\gamma_{r\xi_0}=2\frac{e^2}{c\hbar}\frac{\Omega_0}{n}\frac{p^2_{cv}}{m_0E_{\xi_0}}\pi^2_{l_e,l_h}, \tag{43}$$

where $\Omega_0=|e|H/m_0c$ is the cyclotron frequency for the bare electron mass m_0, n is the index of refraction, the energy E_{ξ_0} is determined in Eq. (1),

$$\pi_{l_e,l_h}=\int_{-\infty}^{\infty}dz\varphi_{cl_e}(z)\varphi_{vl_h}(z), \tag{44}$$

$\varphi_{cl}(z)$, $\varphi_{vl}(z)$ are the real functions, corresponding to the size-quantization number l for the QW of a finite depth (see, for instance, Ref. 6), $c(v)$ is the conduction (valence) band index.

It follows from Eq. (43) that the inverse lifetime $\gamma_{r\xi_0}$ is proportional to H. The dependence on the index n and the magnetic field strength H due to the factor $E_{\xi 0}$ in the denominator is very weak, because in the RHS of Eq. (1) the energy gap is much more than the energy $(n+1/2)\hbar\Omega_\mu$. We suppose that there are no reasons for the strong dependence of γ_ρ on an integer n.

With the help of Eq. (39) we obtain for the reflected flux

$$\mathcal{R}^{sim}(s)=4\left(\frac{\gamma_r}{\gamma_l}\right)^2e^{-\gamma s}Y^{sim}_{\Omega_l,G_l}(S), \tag{45}$$

where

$$Y^{sim}_{\Omega_l,G_l}(S) = \frac{G_l^4}{16}\left\{\left[\sum_{n=0}^{\infty}\frac{\cos(nS)}{(\Omega_l-n)^2+G_l^2/4}\right]^2 + \left[\sum_{n=1}^{\infty}\frac{\sin(nS)}{(\Omega_l-n)^2+G_l^2/4}\right]^2\right\}. \quad (46)$$

The non-dimensional variables $S=\Omega_\mu s$ and

$$\Omega_l = \frac{\omega_l}{\Omega_\mu}, \quad G_l = \frac{\gamma_l}{\Omega_\mu}. \quad (47)$$

are introduced. The function Eq. (46) is periodical one with the period 2π and symmetrical relatively the substitution of S by $2\pi - S$.

In the case of the asymmetrical pulse we obtain instead of Eq. (46) the expression which has a convenient form

$$Y^{asim}_{\Omega_l,G_l}(S) = \frac{G_l^2}{16}[(\sigma_{c0}\Omega_l-\sigma_{c1})^2+(\sigma_{s0}\Omega_l-\sigma_{s1})^2] + \frac{G_l^3}{16}(\sigma_{s1}\sigma_{c0}-\sigma_{s0}\sigma_{c1}) + \frac{G_l^4}{64}(\sigma_{c0}^2+\sigma_{s0}^2), \quad (48)$$

where

$$\sigma_{c0} = \sum_{n=0}^{\infty}\frac{\cos(nS)}{(\Omega_l-n)^2+G_l^2/4},$$
$$\sigma_{c1} = \sum_{n=0}^{\infty}\frac{n\cos(nS)}{(\Omega_l-n)^2+G_l^2/4},$$
$$\sigma_{s0} = \sum_{n=0}^{\infty}\frac{\sin(nS)}{(\Omega_l-n)^2+G_l^2/4},$$
$$\sigma_{s1} = \sum_{n=0}^{\infty}\frac{n\sin(nS)}{(\Omega_l-n)^2+G_l^2/4}. \quad (49)$$

V. A SYMMETRICAL EXCITING PULSE. AN ECHO IN TRANSMITTED AND REFLECTED FLUXES.

In the resonance of the frequency ω_l with a Landau level, i. e. at

$$\Omega_l = n_0, \quad n_0 = 0,1,2,\ldots \quad (50)$$

we obtain from Eq. (46)

$$Y^{sim}_{\Omega_l=n_0,G_l}(S) = \frac{G_l^4}{16} \times\left\{\left[\sum_{n=0}^{\infty}\frac{\cos(nS)}{n^2+G_l^2/4}+\sum_{n=1}^{n_0}\frac{\cos(nS)}{n^2+G_l^2/4}\right]^2 + \left[\sum_{n=n_0+1}^{\infty}\frac{\sin(nS)}{n^2+G_l^2/4}\right]^2\right\}. \quad (51)$$

In the extreme case

$$\Omega_l = n_0, \quad G_l << 1 \quad (52)$$

we obtain from Eq. (51)

$$Y^{sim}_{\Omega_l=n_0,G_l<<1}(S) \simeq 1 + \frac{G_l^2}{2}\left[F_{sim}(S)+\sum_{n=1}^{n_0}\frac{\cos(nS)}{n^2}\right], \quad (53)$$

where

$$F_{sim}(S) = \sum_{n=1}^{\infty}\frac{\cos(nS)}{n^2} = \frac{(\pi-S)^2}{4} - \frac{\pi^2}{16} \quad (54)$$

changes from 0 to 2π. At $G_l \to 0$ we obtain

$$Y_{\Omega_l=n_0,G_l=0} = 1\ldots$$

and, according to Eq. (45),

$$\mathcal{R}^{sim}(s) \simeq 4(\gamma_r/\gamma_l)^2 e^{-\gamma s}\cdots,$$

which corresponds to the contribution from only one LL, which is in the resonance with the frequency ω_l.

Further let us consider the case when the frequency ω_l is in the resonance with one of the upper LLs, i. e.

$$\Omega_l = n_0, \quad n_0 >> 1, \quad (55)$$

and G_l is arbitrary. Then we obtain immediately from Eq. (51)

$$Y^{sim}_{\Omega_l=n_0,n_0>>1,G_l}(S) \simeq \frac{G_l^4}{16}\left(\frac{4}{G_l^2}+2\sum_{n=1}^{\infty}\frac{\cos(nS)}{n^2+G_l^2/4}\right)^2. \quad (56)$$

The sum in the round brackets in the RHS of Eq. (56) is calculated precisely and we obtain

$$Y^{sim}_{\Omega_l=n_0,n_0>>1,G_l}(S) \simeq \frac{G_l^2}{4}\pi^2\cosh^2[(\pi-S)\frac{G_l}{2}] \times cosech^2\left(\frac{\pi G_l}{2}\right). \quad (57)$$

If $G_l << 1$ we obtain from Eq. (57)

$$Y^{sim}_{\Omega_l=n_0,n_0>>1,G_l<<1}(S) \simeq 1 + G_l^2 F_{sim}(S), \quad (58)$$

which accords with Eq. (53) in the limit $n_0 \to \infty$. Under condition

$$G_l >> 1 \quad (59)$$

it follows from Eq. (57)

$$Y^{\ sim}_{\ \Omega_l=n_0,n_0>>1,G_l>>1}(S) \simeq \frac{G_l^2}{4} \times \pi^2\begin{cases} e^{-SG_l}, & S<<1, \\ e^{-(2\pi-S)G_l}, & 2\pi - S << 1. \end{cases} \quad (60)$$

In Figs. 1-4 the curves corresponding to Eq. (46) are represented for the different values of the parameters Ω_l and G_l. The functions $Y^{sim}_{\Omega_l,G_l}(S)$ are periodical ones with the period 2π, only one period is represented. Fig. 1 corresponds to the small value $G_l = 0.1$ and to the values $\Omega_l = 0;\ 0.05;\ 0.1$. At $\Omega_l = 0$ the frequency ω_l is in the resonance with the lower LL for the EHP; the values $\Omega_l = 0.05$ and $\Omega_l = 0.1$ correspond to the small deviation from this resonance. The curves a are calculated on the precise formula Eq. (46) ,the curve b is calculated on the approximate formula Eq. (53). One can conclude from Fig.1 that in the case of the precise resonance of the frequency ω_l with some LL at small values G_l the periodical oscillations of the reflected and transmitted fluxes have very small amplitudes and that the approximate formula Eq. (53) gives the result, which is close to being precise. A small deviation of the frequency ω_l from the resonance results in a sharp drop of the flux values.

Fig. 2 shows the same as Fig. 1, but at the bigger value $G_l = 0.5$. In comparison with Fig. 1 the amplitude of the periodical beatings increases strongly, the approximate formula Eq. (53) works worse , the small deviations of the frequency ω_l from the resonance do not lead to the sharp decrease of the energy fluxes.

Fig. 3 corresponds to the parameter $G_l = 1$ and to the set of values $\Omega_l = 0;\ 0.1;\ 0.5;\ 1.0;\ 1.5;\ 2.0$. In this figure the amplitude of the periodical oscillations of the non-dimensional factor $Y^{sim}_{\Omega_l,G_l}(S)$ reaches the values 1.5 - 2.0. At $\Omega_l = 0.5$ and $\Omega_l = 1.5$, i. e. when the frequency ω_l is between the LLs $n = 0$ and $n = 1$ and between the LLs $n = 1$ and $n = 2$, the curves touch the abscise axis at the point $S = \pi$, thus, the fluxes approach to 0.

Finally, Fig.4 corresponds to the large value $G_l = 5$ and to the set of values : $\Omega_l = 0;\ 0.1;\ 0.5;\ 1.0$. The values of the factor $Y^{sim}_{\Omega_l,G_l}(S)$ at points $S = 0$ and $S = 2\pi$ grow sharply in comparison with the corresponding values in Figs. 1-3, reaching 30, but they become very small in the interval $S >> G_l^{-1}, (2\pi - S) >> G_l^{-1}$. Thus, the periodical function $Y^{sim}_{\Omega_l,G_l}(S)$ is a sequence of the short pulses, displaced with the interval 2π; the duration of every pulse is of order G_l^{-1}. Applying Eq. (45) we obtain that at $\gamma_l >> \Omega_\mu$ in the reflected energy flux there appear echoes of the exciting pulse with the interval $2\pi/\Omega_\mu$, which damp as $e^{-\gamma s}$. According to Eq. (35), the echo would be observable in the transmitted flux also. For the values of Ω_l from 0 to several units , when the frequency ω_l is in the vicinity of the lower LLs, the echo's form is distorted in the pulse replicas. In the resonance of ω_l with the upper LLs $n_0 >> 1$ we obtain, according to Eqs. (35), (60), that the form of the echo-pulses coincides with the exciting pulse form. But the values of these pulses is much smaller due to the small factor $\pi^2(\gamma_r/\Omega_\mu)^2 e^{-\gamma s}$.

VI. AN ASYMMETRICAL EXCITING PULSE.

Considering Eq. (48), we find that this function diverges at points $S = 2\pi m$, because the sum σ_{c1}, defined by Eq. (49), diverges. It means that for an asymmetrical exciting pulse we would obtain the infinite values $Y^{asim}(S)$ in the points $S = 0$ and $S = 2\pi$ in the Figs. 1-4. Of course this conclusion is wrong, because the values $\mathcal{R}$, $\mathcal{A}$, $\mathcal{T}$, defined by Eq. (30), cannot exceed unity. Indeed, the infinite values are cut, but it is difficult to determine the mechanism of this cutting. The approximation Eq. (21) (with the help of which we calculate $\Delta\mathbf{E}_2(p)$ from the RHS of Eq. (19) in the lowest order on the electron-light interaction) is inapplicable in the vicinity $S = 2\pi m$.

Therefore we include here only results, corresponding to the resonant case, when the frequency ω_l coincide with one of the upper Landau levels and the divergences do not appear. At $\Omega_l = n_0, n_o >> 1$ we obtain from Eq. (48)

$$Y^{asim}_{\Omega_l=n_0,n_0>>1,G_l}(S) = \frac{G_l^2}{16}\left[\sum_{n=-\infty}^{\infty}\frac{n\sin(nS)}{n^2+G_l^2/4} + \frac{G_l}{2}\sum_{n=-\infty}^{\infty}\frac{n\cos(nS)}{n^2+G_l^2/4}\right]^2. \quad (61)$$

The sums from the RHS of Eq. (61) are calculated precisely, and we obtain

$$Y^{asim}_{\Omega_l=n_0,n_0>>1,G_l}(S) = \frac{G_l^2}{16}\pi^2\left[cosech(\frac{\pi G_l}{2})\right]^2 e^{(\pi-S)G_l}. \quad (62)$$

In the case $G_l << 1$ we obtain from Eq. (62)

$$Y^{asim}_{\Omega_l=n_0,n_0>>1,G_l<<1}(S) \simeq \frac{1}{4}[1+2G_lF(S)],$$
$$F(S) = \sum_{n=1}^{\infty}\frac{\sin(nS)}{n} = \frac{\pi-S}{2}. \quad (63)$$

The result at $G_l = 0$ is as following

$$Y^{asim}_{\Omega_l=n_0,G_l=0}(S) \simeq \frac{1}{4}, \quad \mathcal{R}^{asim}(s) = (\gamma_r/\gamma_l)^2 e^{-\gamma s}, \quad (64)$$

it corresponds to the resonance of the frequency ω_l with only level in a QW. At $G_l >> 1$ we have

$$Y^{asim}_{\Omega_l=n_0,n_0>>1,G_l>>1}(S) \simeq \frac{\pi^2}{4}G_l^2 e^{-SG_l}. \quad (65)$$

The last result means that in the case of the asymmetrical pulse $\gamma_l >> \Omega_\mu$,i.e. for very short pulses, the exciting pulse echo appears also. Under condition $n_0 >> 1$ the form of the reiterative pulses coincide with the form of the exciting pulse, but the value of the echo-pulse contains

the small factor $\pi^2(\gamma_r/\Omega_\mu)^2 e^{-\gamma s}$, as well as in the case of the symmetrical pulse.

Fig. 5 shows the function $Y^{asim}_{\Omega_l=n_0,n_0>>1,G_l>>1}(S)$ (see Eq. (62)) at the values $G_l = 0.1;\ 1.0;\ 10$. At $G_l = 0.1$ we obtain a saw-like curve and a duplication of the exciting pulse form at $G_l = 10$.

VII. CONCLUSION

Thus, we have calculated the time dependence of the transmitted, reflected and absorbed energy fluxes, appearing under the normal irradiation a QW, subjected to the SMF, by the exciting light pulse. We supposed that the energy levels of the electronic excitations are equidistant with the energy interval $\hbar\Omega_\mu$. The results for the symmetrical and asymmetrical exciting pulses have been obtained. The interrelations between the parameters have been chosen :

$$\gamma_r << \gamma, \quad \gamma << \gamma_l, \quad \gamma << \Omega_\mu, \tag{66}$$

the interrelation of γ_l andΩ_μ is arbitrary one. The energy fluxes have been examined on times

$$t >> \gamma_l^{-1}, \quad t << \gamma_r^{-1}, \tag{67}$$

when the exciting pulse is already damped, the transmitted and reflected fluxes are equal in absolute values, and the absorbed flux is negative and equal in its modulus to the doubled transmitted (or reflected) flux.

The reflected flux contains the factor $e^{-\gamma s}$, which determines its damping. Besides, there is the factor Y, periodical on s with the period $2\pi\Omega_\mu^{-1}$. These oscillations are never sinusoidal ones, characteristically only for the case of two closely displaced excitation energy levels. Under condition $\gamma_l << \Omega_\mu$, i.e. for the comparatively short pulses, which time duration exceeds the value Ω_μ^{-1}, the oscillation amplitude is small. In the limit $\gamma_l/\Omega_\mu = 0$ in the case of the resonance of the frequency ω_l with one of the energy levels, we obtain the results, corresponding to the case of the only energy level. Under condition $\gamma_l \geq \Omega_\mu$ the oscillation amplitude becomes large one. Finally, in the case when $\gamma_l >> \Omega_\mu$, i. e. the pulse duration is much smaller than Ω_μ^{-1}, the damping echo of the exciting pulse appears with the time intervals $2\pi\Omega_\mu^{-1}$.

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†† In the case of the pulse irradiation the lowest approximation on the electron-EMF interaction is acceptable under condition Eq. (2) only on the times $t << \gamma_r^{-1}$, because at $t \geq \gamma_r^{-1}$ the intensity of the transmitted and reflected light damps as $\exp(-\gamma_r t)$.

‡ Eqs. (13)-(15) at $Q(\omega) \simeq 0$, $I(\omega) \simeq 0$ are applicable, if each of the circular polarizations corresponds to one of two types of the EHPs, the energies of which are equal.

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FIG. 1. The function $Y^{sim}_{\Omega_l,G_l}(S)$, corresponding to the periodical factor in the value of the reflected energy flux when a QW is irradiated by the symmetrical light pulse. The pulse duration γ_l^{-1} exceeds on the order the value Ω_μ^{-1}.

FIG. 2. Same as Fig. 1, the pulse duration $\gamma_l^{-1} = 2\Omega_\mu^{-1}$.

FIG. 3. Same as Fig. 1, the pulse duration $\gamma_l^{-1}= \Omega_\mu^{-1}$.

FIG. 4. Same as Fig.1 for the very small value γ_l^{-1} , when the echo of the exciting pulse appears.

FIG. 5. The function $Y^{asim}_{\Omega_l=n_0,n_0>>1,G_l}(S)$ in the case of the asymmetrical exciting pulse with the sharp front. The frequency ω_l is in the resonance with one of the upper Landau levels. The values $G_l = \gamma_l/\Omega_\mu$ are indicated.

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